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Nitrogen Laser Power Density and Some Design Considerations

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14

CONTENTS

ABSTRACT

The laser power density and its time history from a pulsed molecular nitrogen laser were calculated using the theory of Ali, Kolb, and Anderson. The dependence of the power density and its duration on the fill pressure and on various parameters of an electric circuit, in which the nitrogen gas acts as a variable resistor, was examined. It is shown that the peak power density increases in a circuit with low inductance and high capacitor voltage and that the energy density is optimum at a fill pressure of ~30 Torr. Based on these premises and the gain calculations, some design considerations were also studied.

PROBLEM STATUS

This is a final report on one phase of the theoretical calculations on the performance and limitations of high-temperature gas lasers.

AUTHORIZATION

NRL Problem K03-08A
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NITROGEN LASER POWER DENSITY AND SOME DESIGN CONSIDERATIONS

INTRODUCTION

Laser action from the 3371-Å line was first reported by Heard (1). This line corresponds to the $C^3\pi_u \rightarrow B^3\pi_g$ transition in the second positive band of the nitrogen molecule. Since Heard's experiment, many authors have observed this superradiant line under a variety of conditions. The cross-field discharge approach by Leonard (2) and by Shipman (3,4), which produces a large E/P value (where E is the electric field and P is the pressure in Torr), has resulted, respectively, in a 20-kW, 20-nsec output and a 2.5-MW, 4-nsec output. Shipman (5) has increased the power output from one end of his chamber by a factor of ~10 using a traveling wave excitation method. Ericsson and Lidholt (6), on the other hand, have obtained ~2 kW with a very short power duration of ~1 nsec in an experimental setup which is small compared to those used by Leonard (2) and Shipman (3). The fill pressures used have varied from 10 to 30 Torr. However, Svedberg (7) et al. have observed a peak power output of 2 W with power duration of 0.6 nsec in atmospheric pressures with an experimental setup similar to that of Ericsson (6) et al. Svedberg's experiment raises an interesting question as to whether laser action takes place in the lightning discharge. Hopefully this question will be answered in a future paper (8).

Gerry (9) and Ali (10) et al. theoretically have calculated the laser power output and the time history of the laser pulse. Their calculations have predicted results in good agreement with the experiments of Leonard (2) and Shipman (3). These calculations are based on the assumption that laser levels, $C^3\pi_u$ and $B^3\pi_g$, are populated directly from the ground state $X^1\Sigma$ of the molecule by electron impacts. Their recent theoretical calculations (10) take into consideration the effects of collisional mixing of the laser levels by electron impacts and the ionization from the C-state of the molecule, which become significant at increasing electron density. This theory will be used to study the N_2 laser and to set certain requirements on the design and the operation of such a N_2 laser.

BASIC THEORY

The nitrogen molecules, as a lasing system, are coupled to an electric circuit consisting of a capacitor C charged initially to some voltage V_0, a fixed resistance R_e, and an inductance L. The lasing medium, i.e., N_2 molecules, acts as a variable resistance $R(t)$ which is mainly due to electron molecule collisions. The rate equations governing the population densities of the upper and the lower laser levels are

$$\frac{dN_3}{dt} = X_{13}N_1 + X_{23}N_2 - (\tau_{32}^{-1} + Y_{32} + X_{3x})N_3 - R_{32}^{i}(N_3 - N_2) \tag{1}$$

and

$$\frac{dN_2}{dt} = X_{12}N_1 + N_3(\tau_{32}^{-1} + Y_{32}) - (\tau_{21}^{-1} + X_{23})N_2 + R_{32}^{i}(N_3 - N_2), \tag{2}$$

where N_3 and N_2 are, respectively, the population densities of the upper, $C^3\pi_u$, and the lower, $B^3\pi_g$, laser levels. N_1 is the population density of the ground state of the molecule. X_{ij} is the rate of collisional excitation by electron impacts from level i to level j

where $i < j$. Y_{ji} is the rate of collisional de-excitation from j to i, and $(\tau_{ji})^{-1}$ is the rate of the radiative decay from j to i. Finally R'_{32} is the rate of induced emission which includes Einstein's B coefficient, the energy density, and the line width which is considered to be Doppler broadened.

The energy density ρ, at the laser frequency, can be given as

$$(1/E_\ell)\frac{d\rho}{dt} = N_3\tau + R'_{32}(N_3 - N_2), \tag{3}$$

where E_ℓ is the energy of the laser photon. The ionization rate is given by

$$\frac{dN_e}{dt} = N_e N_1 S, \tag{4}$$

where N_e is the electron density and S is the ionization rate coefficient. The electrons are ohmic heated and lose their energies by exciting and ionizing the molecules. This process is given by

$$\frac{d}{dt}\left(\frac{3}{2}N_e kT_e\right) = \gamma I^2 R(t) - N_1(X_{13}E_{31} + X_{12}E_{21}) - N_e N_1 S E_\infty$$
$$- N_e N_1 \overline{X_v E_v} - N_1 X_{3\infty} E_{3\infty} - E_{12}(N_2 X_{21} - N_1 Y_{12}), \tag{5}$$

where E_{ji} are the energies of the $j \to i$ transitions, E_∞ is the ionization energy, k is the Boltzman constant, T_e is the electron temperature, γ is a conversion factor to energy per unit volume, and I is the total current.

Finally, the circuit equation is given by

$$L\frac{dI}{dt} + R_T I = V_0 - \int_0^t \frac{I}{C}\,dt' \tag{6}$$

where $R_T = R_e + R(t)$. The above equations are given here for completeness. They can be found in Refs. 10 and 11, where a discussion is given concerning the rate coefficients and $R(t)$.

LASER POWER CALCULATIONS

The system of Eqs. (1) through (6) can be solved either exactly (10) or by using the saturation approximation method (9,10). The power output and its time history predicted by the latter method (10) agree very well with the results of the exact solution (Fig. 1). Under the saturation assumption, i.e., $N_3 = N_2 = N$ for $N_1 >> (N_3 - N_2)$, the sum and the difference of Eqs. (1) and (2) result in

$$\frac{dN}{dt} = \frac{1}{2}(X_{13} + Y_{12})N_1 - \frac{1}{2}(\tau_{21}^{-1} + Y_{12} + X_{21})N \tag{7}$$

and

$$P = \frac{1}{2}(X_{13} - X_{12})N_1 + (X_{21} + \frac{1}{2}\tau_{21}^{-1} - \tau_{32}^{-1} - Y_{12} - X_{31})N. \tag{8}$$

respectively, where $P = R'_{32}(N_3 - N_2)$. Fewer equations (e.g., Eqs. (4) through (7)) must

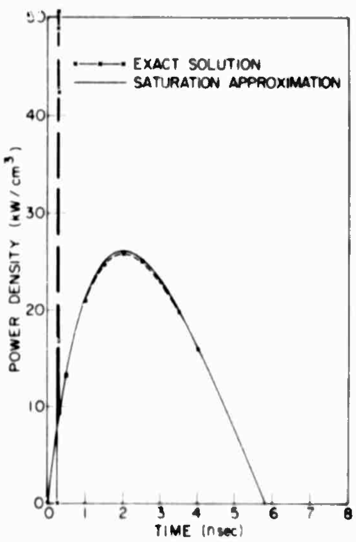


Fig. 1 - A comparison of the exact solution and the saturation approximation method. The power density and its time history predicted by these methods, are in excellent agreement, although the exact solution predicts an early oscillation which is extremely narrow.

be solved to obtain the power density and its time history. Therefore, the saturation approximation method will be used extensively in this report (Fig. 1).

POWER DENSITY AND CURRENT RISETIME

The laser duration and its power density ρ times E_ℓ, where E_ℓ is the energy of the laser photon, depend to a great extent on the current risetime. This is demonstrated in Fig. 2, where the laser power density and its duration are shown for several different values of the circuit inductance. It is obvious from Fig. 2 that the faster the current rises (smaller L), the higher the peak power density becomes and the earlier the laser pulse terminates. For currents with fast risetimes, the ionization rate is higher and the electrons are relatively hotter over the period of interest as shown in Figs. 3 and 4, respectively. The higher rates of ionization imply higher electron densities. The laser power density is essentially proportional to the electron density as shown by Eq. (8), where $N_{12} = N_e \langle \sigma_{12} v \rangle$. Thus, the total energy, i.e., the peak power density times the halfwidth of the laser pulse, is larger for fast rising currents.

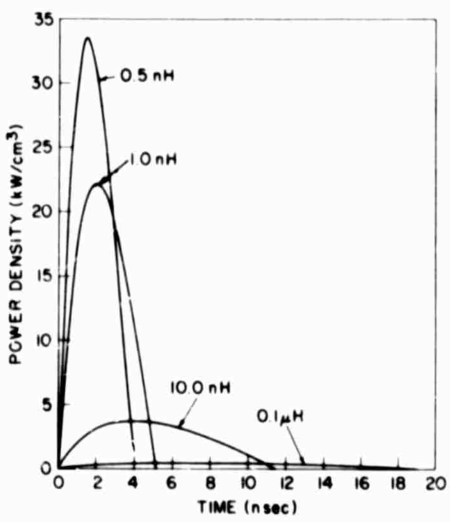


Fig. 2 - The power density as a function of time for several values of the circuit inductance. The peak power density rises with decreasing L values and decreasing current risetimes.

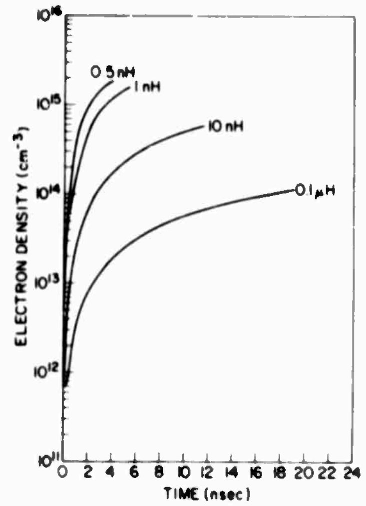


Fig. 3 - Electron density as a function of the current risetime for different values of the circuit inductance

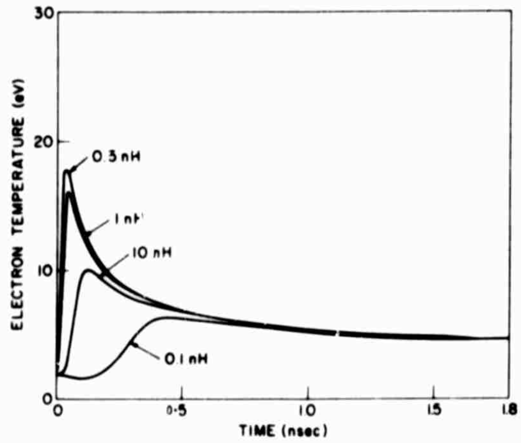


Fig. 4 - The time history of the electron temperature for different values of the circuit inductance

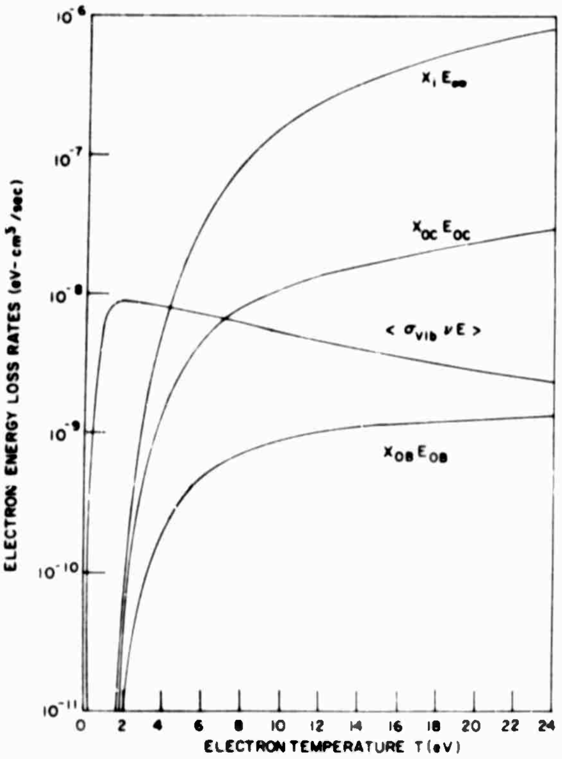


Fig. 5 - Electron energy loss rates as a function of electron temperature

Furthermore, for electrons having temperatures above 6 eV, excitation of the upper laser level is more probable than the excitation of the ground state vibrational levels. This latter process becomes important at low electron temperatures (below 6 eV) and predominates over other rate processes when the electron temperature falls below 4 eV. Figure 5 indicates the rate of the energy loss by electrons to various relevant processes. These rates are obtained by using measured cross sections averaged with the electron velocity over a Maxwellian velocity distribution as found in Ref. 11.

POWER DENSITY AND THE FILL PRESSURE

The laser power density and its time history are shown in Fig. 6 for several values of the initial pressure (fill pressure). It is evident that the peak power density rises with increasing initial pressure for the region between 1 and 30 Torr, and the laser pulse terminates earlier. The energy density has an optimum value at a pressure of 28 Torr in agreement with experimentally

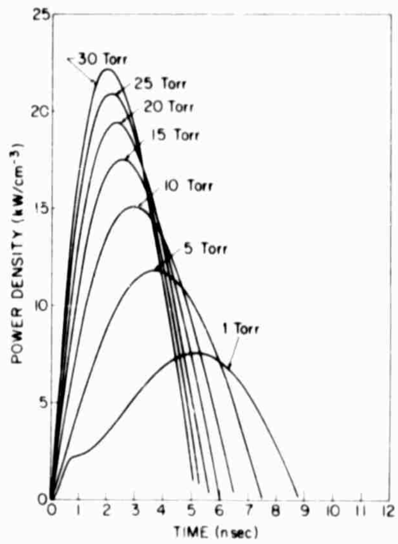


Fig. 6 - Laser power density as a function of time for different fill pressures

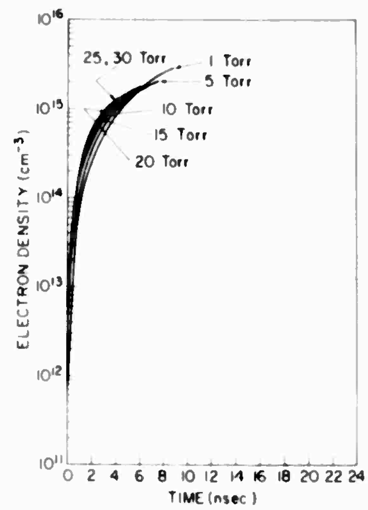
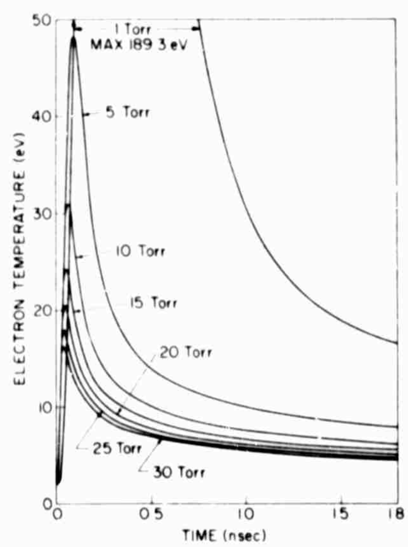


Fig. 7 - Electron density as a function of time for different fill pressures

observed values (2,3) of 25 to 30 Torr. This phenomenon can be explained in conjunction with Figs. 7 and 8, where the electron density and the electron temperature respectively are plotted vs time for several fill pressures. One can estimate the peak power density at 30 Torr to be higher by a factor of 10 compared with the peak power density at 1 Torr. With the assumptions $P \simeq N_e N_1 X_{13}$, N_e (1 Torr) ≈ $N_e/3$ (30 Torr), T_e (1 Torr) ≈ 20 eV, and T_e (30 Torr) ≈ 4 eV, it is estimated that X_{13} (1 Torr) ≈ 10 X_{13} (30 Torr). However, the peak power density decreases and the pulse duration is reduced further with increasing pressure beyond 30 Torr. The previous analysis will not be valid for higher pressures because with increasing pressure, the electrons are cooler (below 4 eV) through losing a considerable amount of their energies to excitation of the ground state vibrational levels.

Fig. 8 - The time history of the electron temperature for different fill pressures



POWER DENSITY AND THE INITIAL CAPACITOR VOLTAGE

The dependence of the laser power density on the initial capacitor voltage V_0 is shown in Fig. 9. The power density is seen to increase with increasing voltage. The intense electric field resulting from the increased voltage leads to higher E/P values which imply higher electron drift velocities. In addition, the higher voltages result in higher electron densities (Fig. 10). Therefore, with higher inputs (ohmic heating per unit volume) into the gas system, higher currents are produced as the E/P and electron density increase. It should also be recognized that the electron temperature is higher with increasing initial voltages, and less energy is lost to ground state vibrational excitations. These two combined effects produce higher peak power densities. Finally, for completeness, the dependence of the power density on the capacitance is illustrated in Fig. 11, where the values of C do not seem to affect the power output.

THE HALFWIDTH OF THE LASER PULSE

In contrast to the previous sections, where the peak power density and its dependence on the circuit parameters were emphasized, in this section the halfwidth of the laser pulse will be discussed and analyzed. Our concern will especially be with the cause of the narrowing of the pulse. The laser line at 3371 Å has a lower level which is long lived compared to its own lifetime (τ_{32} = 40 nsec (12), τ_2 = 10 μsec). This, by itself, would limit the effective lifetime (13) of the upper laser level to

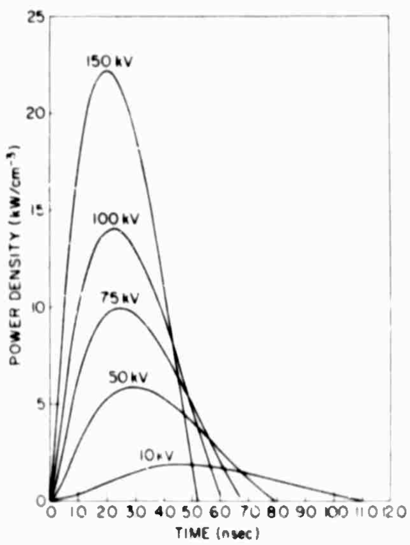


Fig. 9 - Laser power density and its time history for different initial capacitor voltages

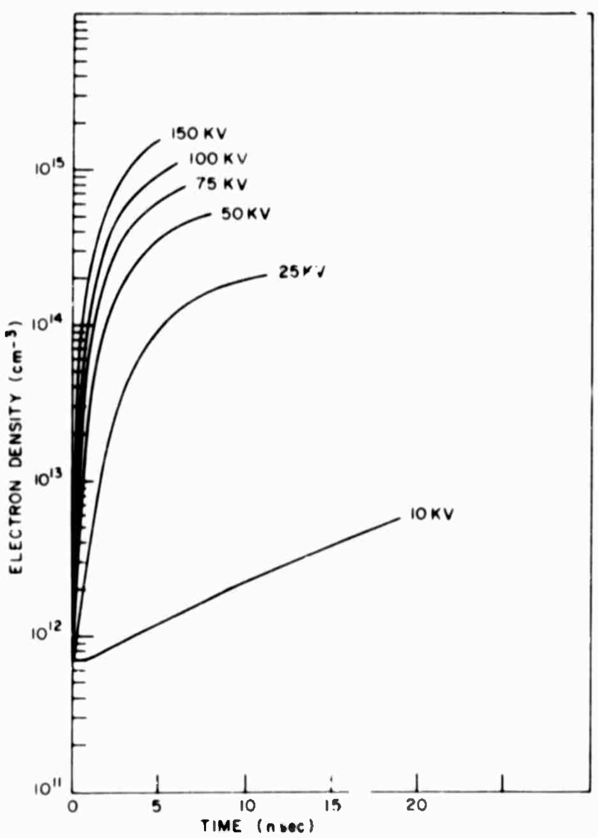
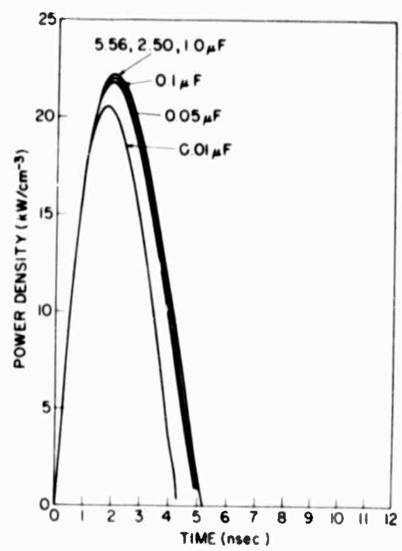


Fig. 10 - The effect of the initial capacitor voltage on the electron density. The electron density can be increased by several orders of magnitude by increasing the voltage.

Fig. 11 - Power density and its time history for different values of capacitance



τ_{32} (effective) $\leq \tau_{32}/2$. However, with increasing laser intensity the induced emission rate (right-hand side of Eq. (3)) predominates over the noise term, and the effective lifetime is practically the inverse of the induced emission rate. In fact, the last term in the right-hand side of Eq. (3) is many orders of magnitude larger than the noise term. Furthermore, two additional factors would influence the population density of the upper laser level at increasing electron density. These are the collisional mixing of the laser levels by electron impacts and the electron impact ionization of the molecule from the c-state. As expected, these two elements will further narrow the duration of the laser pulse. One can, therefore, generalize by saying that whatever causes the increase in the electron density and consequently the laser power density also narrows the laser pulse.

GAIN CALCULATIONS

Let dI denote the increase in the laser intensity along a distance dX. This increase can be expressed as

$$dI = R_{32}^{i}(N_3 - N_2)E_{\ell}\, dX. \tag{9}$$

The same increase or decrease when the gain coefficient α is negative can, in general, be expressed (14) by

$$dI = \alpha\, I\, dX. \tag{10}$$

A comparison of Eqs. (9) and (10) with

$$R_{32}^{i} = \frac{B\rho}{\Delta\nu}$$

leads to

$$\alpha = \frac{R_{32}^{i}}{c} = \frac{BE_{\ell}(N_3 - N_2)}{\Delta\nu\, c}. \tag{11}$$

where c is the velocity of light. Thus the gain in db/m is

$$G_{db/m} = 100\,(10 \log e)\,\alpha\,. \tag{12}$$

Our calculations obtained using the exact solution of the set of Eqs. (1) through (6) indicate that the gain ranges from 2 to 600 db/m at different times depending on the instantaneous current. However, during the period of interest from 0.4 to 2.0 nsec, the gain is in the range of 50 to 100 db/m, which is quite large.

SOME DESIGN CONSIDERATIONS

The implications of the previous sections on the design of the electric circuit is obvious. However, the gain calculations of the last section would imply certain design considerations for the laser chamber. To illustrate, let r denote the radius of the laser chamber with a circular cross section and ℓ denote its length (usually $\ell \gg 2r$). For a gain of 50 db/m, the intensity of the laser radiation, which would spill over the sides along r, can be written as $I = I_0 e^{r/2}$. Therefore with $r = 2$ cm, the laser radiation would be amplified by a factor of 2.7. This will constitute a considerable loss spilled over along the direction of the radius. To confine the amplification along the length of the chamber ℓ, the chamber diameter must be kept small enough to curtail laser radiation spill-over.

The superradiant nature of a nitrogen laser requires a restriction on the diameter of the chamber; however, the restriction on the length of the chamber depends on how the gas is excited. If the cross-field excitation is instantaneous along the entire length of the chamber, the time history of the power density must be considered. For a chamber and electric circuit parameters like Shipman's (3), the laser pulse density terminates in ~5 nsec. This implies that ℓ must be no longer than 150 cm ($\ell = ct$) because the lower laser level is a long lived state (10 μsec) and hence will absorb the laser light. However, if the traveling wave excitation method (5) is utilized, where the excitation along the length of the chamber is synchronized with the arrival of the laser light from the preceding section, then, in principle, one can make ℓ as long as possible.

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Saturation approximation method						
Gain calculations						
Current risetime						
Laser pulse						
Design considerations						
Initial capacitance						
Circuit inductance						